# TOY CASTLE

**Suggested Grades**

$9^{th}$ or $10^{th}$ Grade Geometry

**SD Mathematics Strands & Standards (Primary for Task)**

Measurement

9-12.M.1.3 Students are able to use formulas to find perimeter, circumference, and area to solve problems involving common geometric figures.

Geometry

9-12.G.1.4A Students are able to use formulas for surface area and volume to solve problems involving three-dimensional figures.

**Task Summary**

Students demonstrate their understanding of surface area and volume to calculate the cost of manufacturing a toy castle.

**Time and Context of Task**

If students work alone, this task allows a teacher to assess individual progress. Groups of 2-3 can also be used. 1-2 class periods is all that is necessary to complete this task. If more detail is desired in terms of a student presentation and/or delivery, then more time may be needed. Time needed to complete this task with your students will also depend on the desired level of difficulty you choose to present to your class. This task is designed for use at the end of study on surface area and volume of solids.

**Materials Needed**

Toy Castle Task Worksheet; Paper, Pencil, Calculator

**Author and Lead Teacher for This Task**

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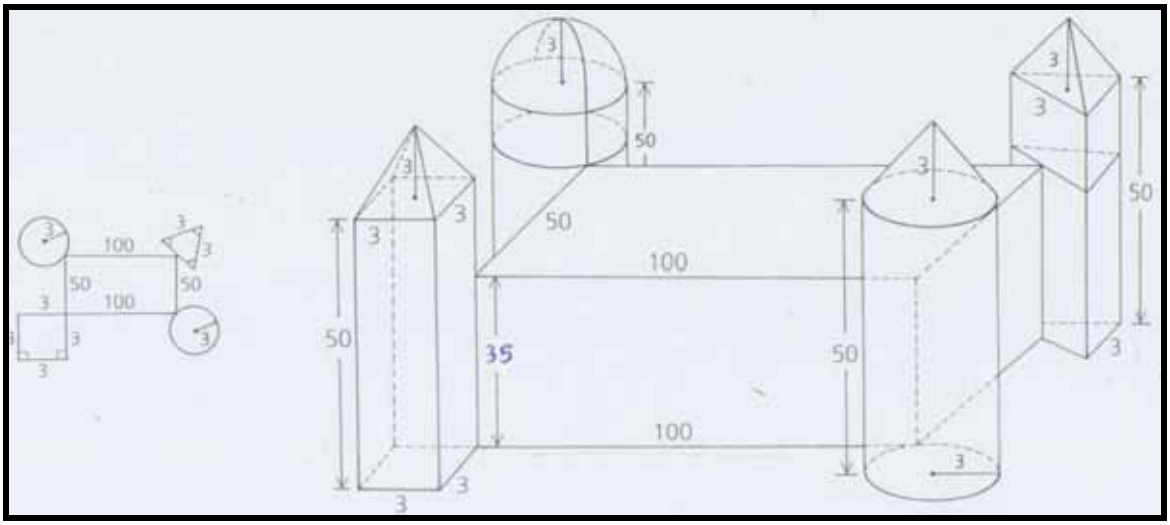
## TOY CASTLE

You are an employee for a major toy manufacturing company. You determine from market research that the newest toys in demand this year will be old renaissance figures. You've decided to design a castle. To aid in convincing the company president that the company should produce such a toy you must first determine the cost of making one toy. In order to calculate the cost of making one toy you must first calculate the total surface area and volume of your castle. The surface area will help determine how much paint would be needed to add color to your toy. The volume will help calculate how much material would be necessary in order to provide dimension to your toy. Your design is shown below in both 3 dimensions and a 2 dimension top view. *All dimensions are given in centimeters*. Using these specifications, prepare a presentation of your calculations for your class.




**Alternative Task**
Divide the castle up into 5 regions and assign each group 2 regions to find the surface area and volume of. This works well if time is an issue.

## CONTENT STANDARDS

**Primary Standards**

**Strand Name: Measurement**

SD Goal: Students will apply systems of measurement and use appropriate measurement tools to describe and analyze the world around them.

Indicator 1: Apply measurement concepts in practical applications.

Standard: 9-12.M.1.3 Students are able to use formulas to find perimeter, circumference, and area to solve problems involving common geometric figures.

**Strand Name: Geometry**

SD Goal: Students will use the language of geometry to discover, analyze, and communicate geometric concepts, properties, and relationships.

Indicator 1: Use deductive and inductive reasoning to recognize and apply properties of geometric figures.

Standard: 9-12.G.1.4A Students are able to use formulas for surface area and volume to solve problems involving three-dimensional figures.

**Supplemental Standard**

**Strand Name**: **Number Sense**

SD Goal: Students will develop and use number sense to investigate the characteristics of numbers in a variety of forms and modes of operation.

Indicator 2: Apply number operations with real numbers and other number systems.

Standard: 9-12.N.2.1 Students are able to add, subtract, multiply, and divide real numbers including integral exponents.

**NCTM Process Standard**

Communication: Use the language of mathematics to express mathematical ideas precisely.

Communication: Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Connections: Recognize and apply mathematics in contexts outside of mathematics.

**Problem-Solving Strategies**

- Estimation and check
- Drawing pictures, graphs, and tables

## ASSESSMENT TOOLS

**Task Rubric**

| | Advanced | Proficient | Basic | Below basic |
|---|---|---|---|---|
| **9-12.M.1.3** Students are able to use formulas to find perimeter, circumference, and area to solve problems involving common geometric figures. | Draws valid and precise conclusions about surface area and volume.<br><br>Student work is accurate, reasonable, complete, and properly labeled. | Draws some valid conclusions about surface area.<br><br>The majority of student work is accurate, reasonable, complete and properly labeled. | Draws valid conclusions about perimeter, circumference, and area of circles, triangles and rectangles.<br><br>Some of the student work is accurate, reasonable, complete and properly labeled. | Draws no conclusion or draws an invalid conclusion.<br><br>Student work shows little evidence of an attempt to find solutions. |
| Communicate Mathematically | Clearly and consistently uses language that is mathematically correct. | Uses language that frequently includes appropriate mathematical terminology. | Uses language that sometimes is mathematically correct. | Uses vague language that does not use mathematical terminology. |
| Convincing Presentation | Presentation shows complete understanding of the mathematical concepts used. It is organized, clear, and convincing. | Presentation shows substantial understanding of the mathematical concepts used. Some organization but not very convincing. | Presentation shows some understanding of the mathematical concepts used. Very little organization. Conclusions are not convincing. | Presentation shows very limited understanding of the underlying concepts needed or no attempt is made to convince. |

**Core High School Geometry**
**Performance Descriptors**

| | |
|---|---|
| **Advanced** | **High school students performing at the advanced level:**<br>• translate and reflect a figure using the coordinate plane;<br>• supply a missing reason and/or statement in a deductive proof. |
| **Proficient** | **High school students performing at the proficient level:**<br>• use deductive reasoning and known properties of a geometric figure to find other properties;<br>• use proportions to solve problems;<br>• translate or reflect a simple figure using the coordinate plane;<br>• match a two-dimensional drawing to its three-dimensional counterpart. |
| **Basic** | **High school students performing at the basic level:**<br>• identify a translation or reflection;<br>• solve a proportion. |

**Core High School Geometry**
**ELL Performance Descriptors**

| | |
|---|---|
| **Proficient** | **High school ELL students performing at the proficient level:**<br>• represent and solve problems involving perimeter, circumference, area, and volume (i.e. volume of a box) of common geometric figures in word problems;<br>• use deductive reasoning and known properties of a geometric figure to find other properties;<br>• use proportions to solve problems;<br>• translate or reflect a simple figure using the coordinate plane;<br>• match a two-dimensional drawing to its three-dimensional counterpart;<br>• read, write, and speak the language of geometry and apply it to geometry problem-solving situations. |
| **Intermediate** | **High school ELL students performing at the intermediate level:**<br>• use geometry concepts;<br>• use formulas to calculate perimeter, circumference, and area of common geometric figures in word problems given oral support;<br>• use geometric terms in explaining solutions orally;<br>• explain in geometric terms the sequence of steps and/or strategies used in problem solving;<br>• give oral, pictorial, symbolic (diagrams) or written responses to questions on topics presented in class. |
| **Basic** | **High school ELL students performing at the basic level:**<br>• calculate perimeter and area of common geometric figures given formulas and figures;<br>• demonstrate problem-solving strategies;<br>• break tasks into smaller parts and make connections to prior knowledge;<br>• recognize, compare and use appropriate geometric terms;<br>• respond to yes or no questions and to problems presented pictorially or numerically in class. |
| **Emergent** | **High school ELL students performing at the emergent level:**<br>• copy and draw geometric shapes and symbols;<br>• imitate pronunciation of geometric terms;<br>• use non-verbal communication to express mathematical ideas such as recognizing simple geometric figures. |
| **Pre-emergent** | **High school ELL students performing at the pre-emergent level:**<br>• observe and model appropriate cultural and learning behaviors from peers and adults;<br>• listen to and observe comprehensible instruction and communicate understanding non-verbally. |

**Core High School Measurement**
**Performance Descriptors**

| | |
|---|---|
| **Advanced** | **High school students performing at the advanced level:**<br>• use dimensional analysis to solve problems;<br>• apply indirect measurement methods;<br>• represent and solve problems involving volume and surface area. |
| **Proficient** | **High school students performing at the proficient level:**<br>• select a suitable unit of measure for problem situations, including rate of change;<br>• choose an appropriate scale for a graph;<br>• represent and solve problems involving perimeter, circumference, and area. |
| **Basic** | **High school students performing at the basic level:**<br>• recognize a unit of measure that describes a rate of change problem;<br>• find circumference and area of circles;<br>• find perimeter and area of rectangles and triangles. |

**Core High School Measurement**
**ELL Performance Descriptors**

| | |
|---|---|
| **Proficient** | **High school ELL students performing at the proficient level:**<br>• use the unit of measurement appropriate for indicated degree of accuracy;<br>• read, write, and speak the language of measurement and apply it to measurement problem-solving situations;<br>• represent and solve problems involving perimeter, circumference, and area. |
| **Intermediate** | **High school ELL students performing at the intermediate level:**<br>• solve measurement problems from oral or written contexts;<br>• use measurement tools and terms to explain the sequence of steps and/or strategies used in solving problems;<br>• give simple oral, pictorial, symbolic (diagrams) or written responses to questions on topics presented in class. |
| **Basic** | **High school ELL students performing at the basic level:**<br>• use measurement tools to solve everyday living problems;<br>• demonstrate problem-solving strategies;<br>• break tasks into smaller parts and make connections to prior knowledge;<br>• recognize, compare, and use appropriate measurement terms;<br>• respond to yes or no questions and to problems presented pictorially or numerically in class. |
| **Emergent** | **High School ELL students performing at the emergent level:**<br>• solve measurement (not word) problems using basic measurement tools;<br>• copy and write measurement symbols;<br>• imitate pronunciation and use of measurement terms;<br>• use non-verbal communication to express mathematical ideas. |
| **Pre-emergent** | **High School ELL students performing at the pre-emergent level:**<br>• observe and model appropriate cultural and learning behaviors from peers and adults;<br>• listen to and observe comprehensible instruction and communicate understanding non-verbally. |

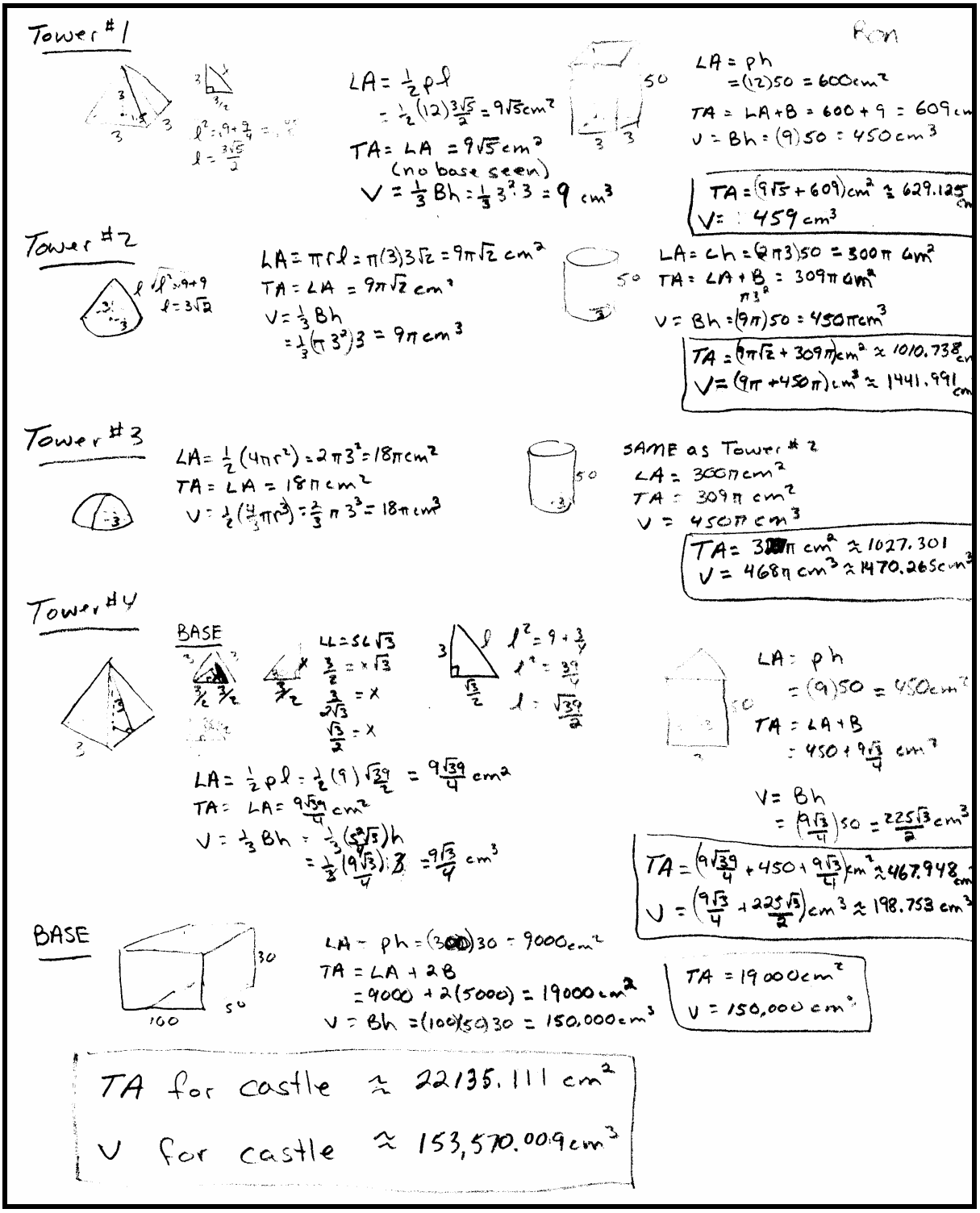
# TOY CASTLE

# Student Work Samples

As you examine the samples, consider the following questions:

- In light of the standard/s addressed and the assessment tools provided, what evidence does the work provide that students are achieving proficiency in the knowledge and skills addressed by the standard/s for the task?
- Is the task/activity well designed to help students acquire knowledge and demonstrate proficiency? Is the task/activity clearly aligned with the standards? In what ways would you adapt the task/activity to better meet the needs of your students?

**Student #1 Work Sample**




**Looking at Student Work – Instructor notes and rating for work sample #1:**

I would rate this student as being advanced. Work is organized, accurate, semi-neat in appearance, and easy to follow. This paper meets the criteria outlined in the advanced column of the assessment rubric.

**Student #2 Work Sample**
**Page 1**

Sphere

$V = \frac{\pi r^2 h}{3}$ $S.A. = \pi r\sqrt{r^2+h^2}$

$V = \pi r^2 h$ $S.A. = 2\pi rh$

B = area of base

Cyl = L.A. $= 2\pi rh$

T.A. $= 2\pi rh + 2\pi r^2$

$V = Bh = \pi r^2 h$

Cone = L.A. $= \frac{1}{2}(2\pi r)\ell = \pi r \ell$

T.A. $= \pi r\ell + \pi r^2$

$V = \frac{1}{3}(\pi r^2)h$

Pyr = L.A. $= \frac{1}{2}p\ell$

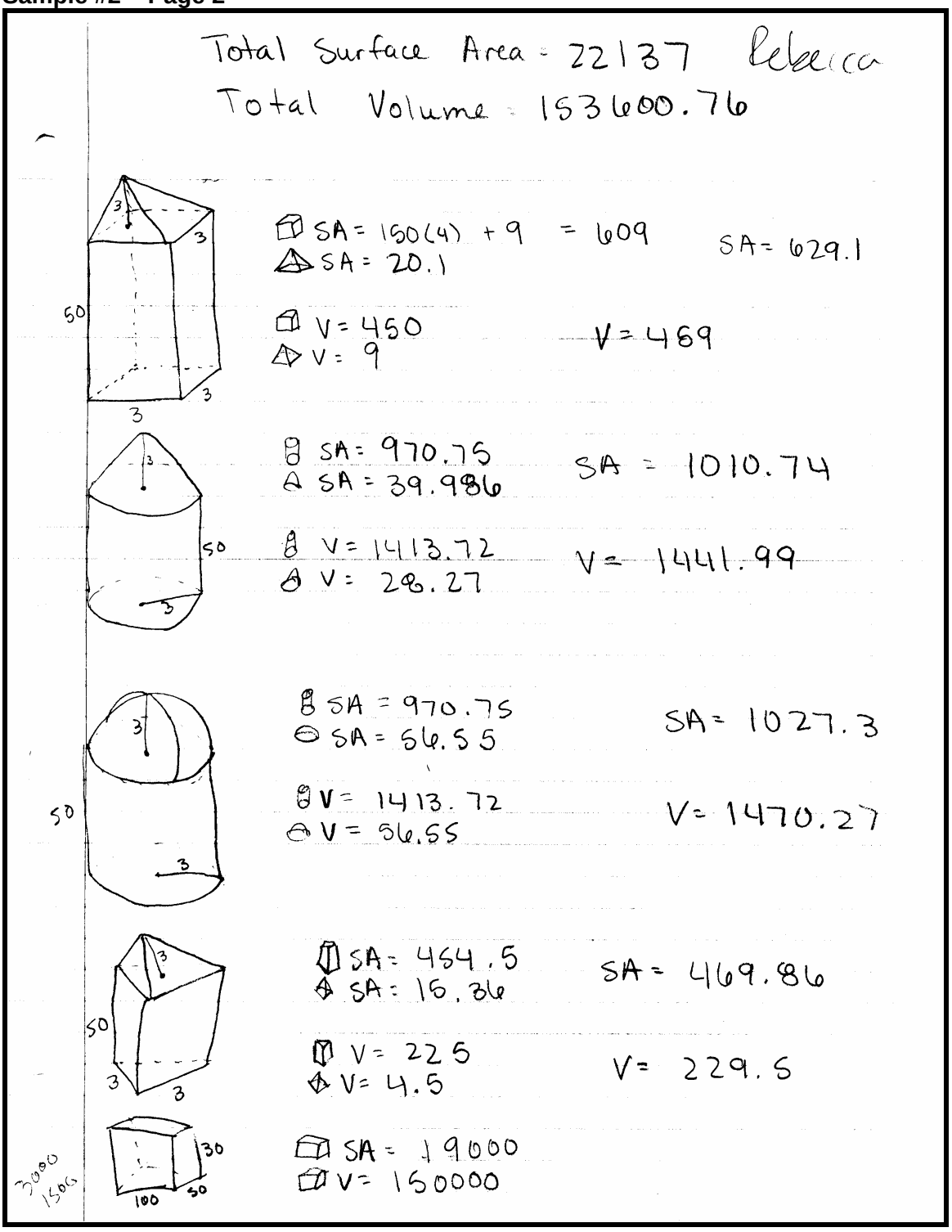
T.A. $= \frac{1}{2}p\ell + B$

$V = \frac{1}{3}Bh$

Sphere $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$SA = \frac{1}{2}(4\pi r^2)$

**Sample #2 – Page 2**




**Looking at Student Work – Instructor notes and rating for work sample #2:**

I would rate this student as proficient based on the assessment rubric. The majority of work is accurate and reasonable. The student used formulas as shown her paper above and chose to show little work which was completed on her graphing calculator. All values are accurate except work shown on the regular triangular pyramid and the right triangular prism.

## INSTRUCTIONAL NOTES

**Author Comments**

To get student samples for this project in a timely manner, this activity was given in the fall to advanced algebra students who had just completed geometry last spring. Normally, students would not encounter this project until the second semester in geometry.

A worksheet that allows students to practice finding surface area and volume of composite figures such as the ones on this performance assessment should be given. I usually assign a worksheet that includes six to seven composite figures to be completed over a period of two days. Most all composite figures assigned have a regular base.

Expectations for the amount of work need to be made clear to all students.

**Task Extension**

Have students develop their own figure out of right prisms, regular pyramids, right cylinders, right cones, and spheres (typical topics covered in geometry class).
OR
Have some dimensions measured in a different unit forcing students to convert to a common unit of measure before proceeding with the task.

**Common Strategies**

A strategy that works well for most all students is to draw each part of the composite figure separately and work to find what is needed side-to-side.
Example: $LA_{cone} = \pi rl$ $LA_{cylinder} = 2\pi rh$

**Common Misunderstandings**

Since this task was given to students not currently enrolled in Geometry some minor mistakes were made. Some students had forgotten what the variables in some of the formulas represented. Some used the wrong formulas altogether in calculating surface area and volume. Volume formulas seemed to be remembered better than surface area formulas, especially in regular pyramids and right cones. Some students forgot that each composite figure would have only one base to include the area of for total area.

**Appropriate Technology**

Any scientific or graphing calculator is appropriate.

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## Resources

**SD Mathematics Content Standards**
http://www.doe.sd.gov/contentstandards/math/index.asp
**SD Assessment and Testing**
http://www.doe.sd.gov/octa/assessment/index.asp
**The National Assessment of Educational Progress (NAEP)**
http://www.doe.sd.gov/octa/assessment/naep/index.asp
**National Council of Teachers of Mathematics**
http://nctm.org/
**Looking at Student Work**
http://www.lasw.org/index.html